The usual (real) octonions are defined by the Cayley-Dickson construction which starts with the quaternion algebra $\mathbb{H}$ (which can itself be obtained from the complex numbers by using the same construction, or from the reals by using it twice) and constructs $\mathbb{O} = \mathbb{H} \oplus \mathbb{H}\ell$ subject to the multiplication rules (for $q, q', r, r' \in \mathbb{H}$) that (0) qq' is the same as in $\mathbb{H}$, (1) $q(r'\ell) = (r'q)\ell$, (2) $(r\ell)q' = (r\bar{q}')\ell$ and (3) $(r\ell)(r'\ell) = -\bar{r}'r$. All of which can be summarized as $(q+r\ell)(q'+r'\ell) = (qq' - \bar{r}'r) + (r'q + r\bar{q}')\ell$. (Mnemonic: if u is a quaternion of unit norm then $q \mapsto q$ and $r\ell \mapsto (ur)\ell$ defines an automorphism of the octonions: this highly constrains the formulæ.) Conjugation is given by $\overline{q + r\ell} = \bar{q} - r\ell$. Using the standard $1, i, j, k$ basis of quaternions, we get the following multiplication table:

$\times$	1	i	j	k	ℓ	$i\ell$	$j\ell$	$k\ell$
1	1	i	j	k	ℓ	$i\ell$	$j\ell$	$k\ell$
i	i	-1	k	$-j$	$i\ell$	$-\ell$	$-k\ell$	$j\ell$
j	j	$-k$	-1	i	$j\ell$	$k\ell$	$-\ell$	$-i\ell$
k	k	j	$-i$	-1	$k\ell$	$-j\ell$	$i\ell$	$-\ell$
ℓ	ℓ	$-i\ell$	$-j\ell$	$-k\ell$	-1	i	j	k
$i\ell$	$i\ell$	ℓ	$-k\ell$	$j\ell$	$-i$	-1	$-k$	j
$j\ell$	$j\ell$	$k\ell$	ℓ	$-i\ell$	$-j$	k	-1	$-i$
$k\ell$	$k\ell$	$-j\ell$	$i\ell$	ℓ	$-k$	$-j$	i	-1

(Line index comes first, so $i \times j = k$.)

This can also be recovered from the rules that (A) for u in $\{i, j, k\}$, the units $u, \ell, u\ell$ obey the same rules as i, j, k (i.e., together with 1, they span a quaternion algebra oriented in this manner), and (B) whenever $u \neq v$ in $\{i, j, k\}$, the octonions u, v, ℓ *anti-associate*, in other words $u(v\ell) = -(uv)\ell$.

The quadratic form $z \mapsto z\bar{z}$ associated to the octonion usual algebra takes $c_1 + c_i i + c_j j + c_k k + c_l \ell + c_{il}\, i\ell + c_{jl}\, j\ell + c_{kl}\, k\ell$ to $c_1^2 + c_i^2 + c_j^2 + c_k^2 + c_l^2 + c_{il}^2 + c_{jl}^2 + c_{kl}^2$.

To construct the split-octonions, we wish to replace ℓ by $\tilde{\ell}$ satisfying $\tilde{\ell}^2 = 1$. One way to do this is to complexify $\mathbb{O}$ and put $\tilde{\ell} = I\ell$ where I is square root of -1 obtained by complexifying. Or one can remove the minus sign in the Cayley-Dickson construction: $(q + r\tilde{\ell})(q' + r'\tilde{\ell}) = (qq' + \bar{r}'r) + (r'q + r\bar{q}')\tilde{\ell}$. Now we have: (A) for u in $\{i, j, k\}$, the units $u, \tilde{\ell}, u\tilde{\ell}$ together with 1 span a split-quaternion algebra oriented in this manner (viz., $u^2 = -1$ but $\tilde{\ell}^2 = (u\tilde{\ell})^2 = 1$), and (B) whenever $u \neq v$ in $\{i, j, k\}$, the split-octonions $u, v, \tilde{\ell}$ *anti-associate*, in other words $u(v\tilde{\ell}) = -(uv)\tilde{\ell}$.

The quadratic form $z \mapsto z\bar{z}$ associated to the split-octonion usual algebra takes $c_1 + c_i i + c_j j + c_k k + c_l \tilde{\ell} + c_{il}\, i\tilde{\ell} + c_{jl}\, j\tilde{\ell} + c_{kl}\, k\tilde{\ell}$ to $c_1^2 + c_i^2 + c_j^2 + c_k^2 - c_l^2 - c_{il}^2 - c_{jl}^2 - c_{kl}^2$.

This description of the split-octonions, however, does not work in characteristic 2. To remedy this, we introduce a different basis, namely $e_0^\pm = \frac{1}{2}(1 \pm \tilde{\ell})$ and $e_1^\pm = \frac{1}{2}(i \pm i\tilde{\ell})$ and $e_2^\pm = \frac{1}{2}(j \pm j\tilde{\ell})$ and $e_3^\pm = \frac{1}{2}(k \pm k\tilde{\ell})$. Then we get the following multiplication table:

$\times$	e_0^+	e_0^-	e_1^+	e_1^-	e_2^+	e_2^-	e_3^+	e_3^-
e_0^+	e_0^+	0	0	e_1^-	0	e_2^-	0	e_3^-
e_0^-	0	e_0^-	e_1^+	0	e_2^+	0	e_3^+	0
e_1^+	e_1^+	0	0	$-e_0^-$	e_3^-	0	$-e_2^-$	0
e_1^-	0	e_1^-	$-e_0^+$	0	0	e_3^+	0	$-e_2^+$
e_2^+	e_2^+	0	$-e_3^-$	0	0	$-e_0^-$	e_1^-	0
e_2^-	0	e_2^-	0	$-e_3^+$	$-e_0^+$	0	0	e_1^+
e_3^+	e_3^+	0	e_2^-	0	$-e_1^-$	0	0	$-e_0^-$
e_3^-	0	e_3^-	0	e_2^+	0	$-e_1^+$	$-e_0^+$	0

(Along with the unit $1 = e_0^+ + e_0^-$, and conjugation taking $e_0^\pm$ to $e_0^\mp$ and $e_n^\pm$ to $-e_n^\pm$ for $n \in \{1, 2, 3\}$.)

Now the quadratic form takes $c_{0\sharp}\, e_0^+ + c_{0\flat}\, e_0^- + c_{1\sharp}\, e_1^+ + c_{1\flat}\, e_1^- + c_{2\sharp}\, e_2^+ + c_{2\flat}\, e_2^- + c_{3\sharp}\, e_3^+ + c_{3\flat}\, e_3^-$ to $c_{0\sharp}c_{0\flat} + c_{1\sharp}c_{1\flat} + c_{2\sharp}c_{2\flat} + c_{3\sharp}c_{3\flat}$.

We can define derivations $\delta_1, \delta_2, \delta_3, \rho_1^\pm, \rho_2^\pm, \rho_3^\pm, \lambda_1^\pm, \lambda_2^\pm, \lambda_3^\pm$ as follows:

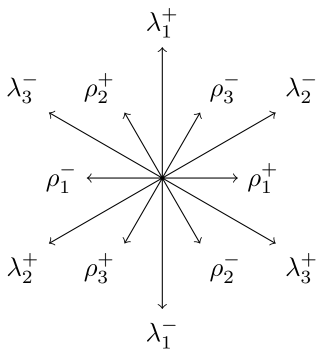
	e_0^+	e_0^-	e_1^+	e_1^-	e_2^+	e_2^-	e_3^+	e_3^-
δ_1	0	0	0	0	e_2^+	$-e_2^-$	$-e_3^+$	e_3^-
δ_2	0	0	$-e_1^+$	e_1^-	0	0	e_3^+	$-e_3^-$
δ_3	0	0	e_1^+	$-e_1^-$	$-e_2^+$	e_2^-	0	0
ρ_1^+	$-e_1^+$	e_1^+	0	$-e_0^+ + e_0^-$	e_3^-	0	$-e_2^-$	0
ρ_1^-	e_1^-	$-e_1^-$	$e_0^+ - e_0^-$	0	0	e_3^+	0	$-e_2^+$
ρ_2^+	$-e_2^+$	e_2^+	$-e_3^-$	0	0	$-e_0^+ + e_0^-$	e_1^-	0
ρ_2^-	e_2^-	$-e_2^-$	0	$-e_3^+$	$e_0^+ - e_0^-$	0	0	e_1^+
ρ_3^+	$-e_3^+$	e_3^+	e_2^-	0	$-e_1^-$	0	0	$-e_0^+ + e_0^-$
ρ_3^-	e_3^-	$-e_3^-$	0	e_2^+	0	$-e_1^+$	$e_0^+ - e_0^-$	0
λ_1^+	0	0	0	0	0	e_3^-	$-e_2^+$	0
λ_1^-	0	0	0	0	e_3^+	0	0	$-e_2^-$
λ_2^+	0	0	$-e_3^+$	0	0	0	0	e_1^-
λ_2^-	0	0	0	$-e_3^-$	0	0	e_1^+	0
λ_3^+	0	0	0	e_2^-	$-e_1^+$	0	0	0
λ_3^-	0	0	e_2^+	0	0	$-e_1^-$	0	0

Evidently $\delta_1 + \delta_2 + \delta_3 = 0$ — all others are linearly independent and span the 14-dimensional Lie algebra $\mathfrak{g}_2$ of derivations of the split-octonions.

Lie bracket table:

	Cartan			Short roots						Long roots					
$[,]$	δ_1	δ_2	δ_3	ρ_1^+	ρ_1^-	ρ_2^+	ρ_2^-	ρ_3^+	ρ_3^-	λ_1^+	λ_1^-	λ_2^+	λ_2^-	λ_3^+	λ_3^-
δ_1	0	0	0	0	0	ρ_2^+	$-\rho_2^-$	$-\rho_3^+$	ρ_3^-	$2\lambda_1^+$	$-2\lambda_1^-$	$-\lambda_2^+$	λ_2^-	$-\lambda_3^+$	λ_3^-
δ_2	0	0	0	$-\rho_1^+$	ρ_1^-	0	0	ρ_3^+	$-\rho_3^-$	$-\lambda_1^+$	λ_1^-	$2\lambda_2^+$	$-2\lambda_2^-$	$-\lambda_3^+$	λ_3^-
δ_3	0	0	0	ρ_1^+	$-\rho_1^-$	$-\rho_2^+$	ρ_2^-	0	0	$-\lambda_1^+$	λ_1^-	$-\lambda_2^+$	λ_2^-	$2\lambda_3^+$	$-2\lambda_3^-$
ρ_1^+	0	ρ_1^+	$-\rho_1^+$	0	$\delta_2 - \delta_3$	$-2\rho_3^-$	$3\lambda_3^+$	$2\rho_2^-$	$-3\lambda_2^-$	0	0	ρ_3^+	0	0	$-\rho_2^+$
ρ_1^-	0	$-\rho_1^-$	ρ_1^-	$-\delta_2 + \delta_3$	0	$3\lambda_3^-$	$-2\rho_3^+$	$-3\lambda_2^+$	$2\rho_2^+$	0	0	0	ρ_3^-	$-\rho_2^-$	0
ρ_2^+	$-\rho_2^+$	0	ρ_2^+	$2\rho_3^-$	$-3\lambda_3^-$	0	$-\delta_1 + \delta_3$	$-2\rho_1^-$	$3\lambda_1^+$	0	$-\rho_3^+$	0	0	ρ_1^+	0
ρ_2^-	ρ_2^-	0	$-\rho_2^-$	$-3\lambda_3^+$	$2\rho_3^+$	$\delta_1 - \delta_3$	0	$3\lambda_1^-$	$-2\rho_1^+$	$-\rho_3^-$	0	0	0	0	ρ_1^-
ρ_3^+	ρ_3^+	$-\rho_3^+$	0	$-2\rho_2^-$	$3\lambda_2^+$	$2\rho_1^-$	$-3\lambda_1^-$	0	$\delta_1 - \delta_2$	ρ_2^+	0	0	$-\rho_1^+$	0	0
ρ_3^-	$-\rho_3^-$	ρ_3^-	0	$3\lambda_2^-$	$-2\rho_2^+$	$-3\lambda_1^+$	$2\rho_1^+$	$-\delta_1 + \delta_2$	0	0	ρ_2^-	$-\rho_1^-$	0	0	0
λ_1^+	$-2\lambda_1^+$	λ_1^+	λ_1^+	0	0	0	ρ_3^-	$-\rho_2^+$	0	0	$-\delta_1$	λ_3^-	0	$-\lambda_2^-$	0
λ_1^-	$2\lambda_1^-$	$-\lambda_1^-$	$-\lambda_1^-$	0	0	ρ_3^+	0	0	$-\rho_2^-$	δ_1	0	0	λ_3^+	0	$-\lambda_2^+$
λ_2^+	λ_2^+	$-2\lambda_2^+$	λ_2^+	$-\rho_3^+$	0	0	0	0	ρ_1^-	$-\lambda_3^-$	0	0	$-\delta_2$	λ_1^-	0
λ_2^-	$-\lambda_2^-$	$2\lambda_2^-$	$-\lambda_2^-$	0	$-\rho_3^-$	0	0	ρ_1^+	0	0	$-\lambda_3^+$	δ_2	0	0	λ_1^+
λ_3^+	λ_3^+	λ_3^+	$-2\lambda_3^+$	0	ρ_2^-	$-\rho_1^+$	0	0	0	λ_2^-	0	$-\lambda_1^-$	0	0	$-\delta_3$
λ_3^-	$-\lambda_3^-$	$-\lambda_3^-$	$2\lambda_3^-$	ρ_2^+	0	0	$-\rho_1^-$	0	0	0	λ_2^+	0	$-\lambda_1^+$	δ_3	0

With respect to the Cartan algebra $\mathfrak{h} := \langle \delta_1, \delta_2, \delta_3 \rangle$, the root system is as follows:



(Note that $e_1^\pm, e_2^\pm, e_3^\pm$ are weight vectors having the same weights as the short roots $\rho_1^\pm, \rho_2^\pm, \rho_3^\pm$.)

If L_x and R_x stand for left- and right-multiplication by the split-octonion x and we let $D_{x,y} = [L_x, L_y] + [L_x, R_y] + [R_x, R_y]$, then this defines a derivation for all x, y, as given by the following table:

$x\downarrow$ $D_{x,y}$ $y\rightarrow$	e_0^+	e_0^-	e_1^+	e_1^-	e_2^+	e_2^-	e_3^+	e_3^-
e_0^+	0	0	ρ_1^-	$-\rho_1^+$	ρ_2^-	$-\rho_2^+$	ρ_3^-	$-\rho_3^+$
e_0^-	0	0	$-\rho_1^-$	ρ_1^+	$-\rho_2^-$	ρ_2^+	$-\rho_3^-$	ρ_3^+
e_1^+	$-\rho_1^-$	ρ_1^-	0	$-\delta_2 + \delta_3$	ρ_3^+	$3\lambda_3^-$	$-\rho_2^+$	$-3\lambda_2^+$
e_1^-	ρ_1^+	$-\rho_1^+$	$\delta_2 - \delta_3$	0	$3\lambda_3^+$	ρ_3^-	$-3\lambda_2^-$	$-\rho_2^-$
e_2^+	$-\rho_2^-$	ρ_2^-	$-\rho_3^+$	$-3\lambda_3^+$	0	$\delta_1 - \delta_3$	ρ_1^+	$3\lambda_1^-$
e_2^-	ρ_2^+	$-\rho_2^+$	$-3\lambda_3^-$	$-\rho_3^-$	$-\delta_1 + \delta_3$	0	$3\lambda_1^+$	ρ_1^-
e_3^+	$-\rho_3^-$	ρ_3^-	ρ_2^+	$3\lambda_2^-$	$-\rho_1^+$	$-3\lambda_1^+$	0	$-\delta_1 + \delta_2$
e_3^-	ρ_3^+	$-\rho_3^+$	$3\lambda_2^+$	ρ_2^-	$-3\lambda_1^-$	$-\rho_1^-$	$\delta_1 - \delta_2$	0