On the Microlocal Regularity
of Very Ample Invertible Sheaves
on a Structured Locally Hyperconvex
Ramanujan Manifold
(and on the meaning of life)

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Abstract: We (almost manage to) prove a few results that could possibly extend to give an intuitive motivation for certain cases of the Schopenhauer-Deligne conjecture on the microlocal regularity of very ample invertible sheaves on a structured locally hyperconvex Ramanujan manifold, at least in the subparacompact limit. Together with this major breakthrough, we also achieve a few minor results, such as proving the unprovability of the Riemann hypothesis, calculating the mass of the neutrino, proving the existence of God, finding the meaning of life (the universe, and everything), explaining why the grass is always greener on the other side of the fence, and giving the scarecrow a brain ([Bau]).

1. Preliminary lemmas.

We start by stating the central lemma, whose proof will occupy most of section 2:

Lemma (1.1). *42 is not a prime number.*

We will also need the following easy results:

Lemma (1.2). *Let $\mathfrak{E}$ be a Grothendieck-Archimedes hypertopos ([Gro]) with internal quantifiers $\exists$, $\dashv$ and $\forall$; then any locally regular D-structor on $\mathfrak{E}$ that is fundamentally based and admits arbitrarily large homotopy types is embedded in a sparse type VIb von Neumann meta-algebra with cherries on top.*

Proof: Trivial. ♡

Lemma (1.3). *If there exists $A_{\mathrm{wiz}}(A_{\mathrm{wiz}})$, then the* w.w. *of $O(z)$ is one.*

Proof: Because the $\{t\colon \mathrm{he} \Vdash t\}$ is w.-full. ♡

Notice that lemma (1.3) has the following immediate corollary:

Corollary (1.4). *There exists, up to isomorphism, exactly one $H_0(M_e)$.*

Lemma (1.5). *All crows are black.*

Proof: Who needs it? It's just a bunch of stupid birds, anyway. ♡

Lemma (1.6). *Socrates is mortal.*

Proof: The lemma follows from the following three statements:

(i) All men are mortal.
(ii) Socrates is a man.
(iii) The demand for very ample invertible sheaves has increased dramatically on the Wall Street market yesterday.

Statement (i) is well-known. Statement (ii) is left as a trivial exercise for any enterprising Athenian youth. As for (iii), it is just a word from our sponsor, included here for completeness. ♡

The above proof is due (with a slight variation) to Aristotle. We note that professor Hemlock of the university of Sparta has proposed a much shorter proof of lemma (1.6) (see [Pla]). Despite its ingenuous use of analytic methods, we shall not present it here, as there is some doubt as to its validity.

Lemma (1.7). *The universe is expanding.*

Proof: It took me almost an hour to go to the chemist last Tuesday. Now apply lemma (1.1). ♡

2. Proof of the central lemma.

We now proceed to prove lemma (1.1). Some intuitive arguments (such as "42 is pretty damn close to 6 times 7", "in this universe, $42 = 6 \times 9$", "I'd say it's well past its prime now" or "what kind of fascist are you, anyway") have been given ([Ada]), but since we intend on doing things fully, we will give a rigorous proof.

Let $(X, \mathcal{O})$ be a locally ringed space, and $\mathcal{F}$ a coherent sheaf of $\mathcal{O}$-modules.

We consider an open subset U of X, and $j: U \hookrightarrow X$ the canonical map. Then we have a sequence of adjoint functors

$$j^\sharp \dashv j_\natural \dashv j^\flat \dashv j_! \dashv j^* \dashv j_* \dashv j^! \dashv j_\clubsuit \dashv j^\spadesuit \dashv j_\heartsuit \dashv j^\diamondsuit$$

and a similar sequence for the inclusion map i of $Y = X \setminus U$ in X. We have canonical maps

$$0 \to j^\sharp j_\heartsuit \mathcal{F} \to \mathcal{F} \to j_\natural j^\diamondsuit \mathcal{F} \to 0$$

that is natural in $\sharp$ and $\flat$ and is exact.

Taking the tensor product of the above sequence with a test $\mathcal{O}$-module $\mathcal{E}$, we get the long exact sequence

$$\cdots \to \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{E}, j_\natural j^\diamondsuit \mathcal{F}) \to \mathcal{E} \otimes_{\mathcal{O}} j^\sharp j_\heartsuit \mathcal{F} \to \mathcal{E} \otimes_{\mathcal{O}} \mathcal{F} \to \mathcal{E} \otimes_{\mathcal{O}} j_\natural j^\diamondsuit \mathcal{F} \to 0$$

This exact sequence is functorial in the sense that if we have a morphism $\mathcal{E} \to \mathcal{E}'$, then the diagram

$$\begin{array}{ccccccccccc}
\cdots & \to & \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{E}, j_\natural j^\diamondsuit \mathcal{F}) & \to & \mathcal{E} \otimes_{\mathcal{O}} j^\sharp j_\heartsuit \mathcal{F} & \to & \mathcal{E} \otimes_{\mathcal{O}} \mathcal{F} & \to & \mathcal{E} \otimes_{\mathcal{O}} j_\natural j^\diamondsuit \mathcal{F} & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\cdots & \to & \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{E}', j_\natural j^\diamondsuit \mathcal{F}) & \to & \mathcal{E}' \otimes_{\mathcal{O}} j^\sharp j_\heartsuit \mathcal{F} & \to & \mathcal{E}' \otimes_{\mathcal{O}} \mathcal{F} & \to & \mathcal{E}' \otimes_{\mathcal{O}} j_\natural j^\diamondsuit \mathcal{F} & \to & 0
\end{array}$$

(this is all obvious, except the last square, which is left as an exercise to the reader).

If we now introduce the corresponding multifunctor for i, then we get a four-dimensional commutative diagram which we shall not bother to write down for it is too simple. Note however that its hyperrows are exact, and if we break them down in short exact sequences and use the multi-snake (also called "king cobra") lemma, then we get a five-dimensional commutative diagram. If we furthermore let U vary (coherently, regularly, flatly and semi-locally), then we get a natural transformation from the obvious functor to the even more obvious functor, together with a six-dimensional commutative diagram.

If we consider the category of such natural transformations (with the obvious morphisms), and we let $\mathbf{C}$ be the slice category of *that* category over the object associated to $\mathcal{G}$ in the obvious manner ($\mathcal{G}$ an arbitrary, contra-test $\mathcal{O}$-module), then the category $\mathbf{C}$ is clearly Cartesian closed, and a moment's reflection suffices to show that it possesses a subobject classifier. It is therefore an elementary topos. We let $j_{\mathcal{F}}$ be its Lawvere-Tierney topology.

We consider the universal such $j_{\mathcal{F}}$, and call it j_∞. Similarly, we consider the couniversal such $j_{\mathcal{F}}$ and call it j_0. Then we have two canonical morphisms

$$j_\infty \rightrightarrows j_0$$

and we let e be their equalizer. It is obvious that proving the lemma reduces to showing that e^{42} is not an isomorphism for some $(X, \mathcal{O})$. But using the Nietzsche-Davenport criterion of purity ([Nie]), together with lemma (1.2), this results follows immediately from the vanishing of the ramassé cohomology groups for $\tilde{X}$, and one can certainly find such X. QED. ♡

3. Local, very local, microlocal, hyperlocal, and low call analysis of a sheaf.

We now let $\mathfrak{M}$ be a Ramanujan manifold, which we will assume is locally hyperconvex except in the first proposition. We let Ω be a partial differential equation, that is no more than a one hundredth part of a differential equation, on $\mathfrak{M}$. We let $\mathcal{F}$ be the sheaf of solutions to Ω, and A (read as "alpha") the ideal equation satisfied by $\mathcal{F}$. We recall the following proposition, taken from [God]:

Proposition (3.1). $\mathrm{A} = \Omega$.

(In fact, in [God], a little more is proven, since both A and Ω are shown to be equal to the author.)

From now on, we assume $\mathfrak{M}$ is locally hyperconvex.

Proposition (3.2). *For all $\varepsilon > 0$ there exists $\delta > 0$ such that for all $\eta > 0$ and $\mu > 0$ there exists $\nu > 0$ such that if $\tau > 0$ then we have $|f(x) - f(x_0) - \rho(x, x_0)| \leq \varepsilon + \eta\tau$ if $|x - x_0| < \delta\tau$ and $|\rho(x, x_0)| < \nu$ for any hyperweak solution f of Ω in the neighborhood of x_0, and moreover the neighborhood in question contains the ball of radius at least μ around x_0. (In all this, ρ is the hyperregulator of the equation.)*

Proof: This follows immediately from proposition (3.1), the local hyperconvexity, and lemma (1.7) together with the following estimate:

$$1 < \sqrt{\pi} 2^{2^{1729!}!}!$$

which we leave as a (difficult) exercise for the interested reader (the hard part, is, of course, getting $<$ rather than just $\leq$), or see [Har]. ♡

Corollary (3.3). *The sheaf $\mathcal{F}$ is microlocally regular.*

Proof: Apply the previous proposition with $\eta = \delta/6$, $\mu = \delta/183$, and $\tau = \nu\delta^{5/2}$, then let $\varepsilon \to 0$. ♡

Corollary (3.4). *The cohomology groups of $\mathfrak{M}$ with coefficients in $\mathcal{F}$ all vanish.*

Proof: This follows immediately from corollary (3.3). ♡

We are therefore led to formulate the following conjecture, which implies the Schopenhauer-Deligne conjecture (see [Nie]):

Conjecture (3.5). *Any very ample invertible sheaf on a structured locally hyperconvex Ramanujan manifold is as above.*

We do not have a full proof of the conjecture. We do, however, have a proof in the following important case:

Case (3.6). *If the sheaf $\mathcal{F}$ is free, then conjecture (3.5) is correct.*

Proof: Trivial. ♡

The above result was the core of this article. We now turn to the more frivolous matters stated in the abstract.

4. A few unimportant things.

Theorem (4.1). *The statement "the non trivial zeros of the Riemann ζ function all lie on the axis $\sigma = \frac{1}{2}$" cannot be proven (relative to the existence of a woppingly large cardinal). The statement "for every number field K, the non trivial zeros of ζ_K all lie on the axis $\sigma = \frac{1}{2}$" cannot be falsified.*

Proof: Let us prove the first statement. Let κ be a woppingly large cardinal, and let S be a Souslin line. Then it is clear that the metaspace $\mathfrak{M}$ of S^κ is a structured locally hyperconvex Ramanujan manifold, and if we let $\mathcal{F}$ be the constant sheaf $\mathbb{Z}$ on $\mathfrak{M}$ then it is clear from proposition (3.1) that $\mathcal{F}$ satisfies the hypothesis of section 3, and therefore satisfies corollary (3.4). This implies the Riemann hypothesis, as is well known.

To get a proof of the second statement, take the above proof, read it backwards, and replace each word by the next word in the Oxford new English dictionary. ♡

Theorem (4.2). *The mass of the neutrino is an element of* $\{0, 1\}$ *(in the appropriate units).*

Proof: It is well known that no mathematician can count beyond that. Now apply lemma (1.1) and proposition (`Runtime error: numeric overflow`). ♡

Theorem (4.3). *God exists.*

Proof: Trivial. Where else would such wonderful results come from? See also [Nie]. ♡

Theorem (4.4). *The meaning of life is summarized by the equation*

$$0 = 0$$

Proof: Apply theorem (4.3) together with proposition (3.1), lemma (1.1), lemma (1.6) and lemma (1.7). ♡

Theorem (4.5). *Grass is always greener on the other side of the fence.*

Proof: The following diagram commutes

$$\begin{array}{ccccc} \text{side} & \rightarrow & \textit{fence} & \rightarrow & {}^{\text{other}}\text{side} \\ \downarrow & & \downarrow & & \downarrow \\ \text{grass} & = & \text{grass} & = & \text{grass} \end{array}$$

and apply all the previous results. ♡

Theorem (4.6). *How to give the scarecrow a brain and get Dorothy back to Kansas.*

How: Follow the yellow brick road, apply lemma (1.3), get the broomstick of the wicked witch of the west and (for the second part) apply corollary (1.4). ♡

5. Bibliography.

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[Pla] Plato, *Phædo*, Athens Academic Press.